

CHAPTER 4

- ▣ The idea for chapter 4 came from the Masters thesis of my student Janine du Plessis, from Georgia State University,
- ▣ **TRANSFORMATION GROUPS AND DUALITY IN THE ANALYSIS OF MUSICAL STRUCTURE.**

Aware that the first contact with a potential reader would be the abstract of her thesis, the abstract was developed with the purpose of:

- ▣ Making the possible reader aware that there is an alternative way to relate Mathematics and Music
1. Traditionally, the study of pitch intervals was done using frequency ratios of the powers of small integers.
 2. Modern mathematical music theory offers an independent way of understanding the pitch system by considering intervals as transformations.

3. Taking advantage of the historical emergence of algebraic structures in musicology and, in the spirit of *Transformational theory*, studying operations that form mathematical groups.
4. Exploring and developing aspects of Neo-Riemannian theory, in particular the T/I and PLR groups as duals (an idea that came from the article **Musical Actions of Dihedral Groups** by Crans, Fiore y Satyendra, available in the ArXiv).

5. Presenting the tools of Group Theory for musical analysis and, above all, providing detailed proofs of many claims that are proposed but seldom formalized in demonstrations.

It was this last point that sparked the idea of augmenting and enhancing Dr. Lluís-Puebla's textbook on Group Theory, to include examples of musical applications in the first three chapters and a fourth chapter.

- ▣ This fourth chapter, in a smooth and “friendly” way, would explain some basic aspects of Mathematical Music Theory;
- ▣ In the process, it would give needed information to readers with different backgrounds, both in Mathematics and in Music.

- ▣ The examples of chapter 4 follow from some of the most important theoretical aspects of the previous chapters;
- ▣ The musical objects and terms are introduced as they are needed, so that a reader without musical background can understand the essence of how Group Theory is employed to explain certain established musical relations.
- ▣ In a similar way, for the reader with knowledge of Music Theory, this chapter provides concrete elements of Group Theory, as well as motivation for understanding it.

- ▣ In terms of antecedents, we can mention the texts on Post Tonal Analysis that are used, for example, in Music Departments and where, as in all applications, Group Theory is presented in an implicit manner.
- ▣ The classic **The Fascination of Groups** by **Budden**, which offers examples from Music and Bell Ringing, together with exercises, but that, by no means tries to be formal, as the author clarifies in the preface.
- ▣ The chapter on Symmetries in Music from **Mathematics a Musical Offering** by **Benson**, also gives a nice panorama of applications and excellent exercises.

What, we think, distinguishes chapter 4 of this text is that it :

- ▣ Intends to take some fundamental notions from **Group Theory**, presented in a formal manner in the first three chapters, and see how certain pre-established musical relations can be explained in these terms.
- ▣ Some aspects of **Transformational and Neo-Riemannian Theory** that have arisen in the last 30 years of academic study of the subject are used as well, giving them a more formal treatment than what the nature of other works on the subject permits.

- ▣ The **exercises** require a lot of detail, hints are given and the complete answers are available.

Indeed, many of the exercises consist of:

- ▣ **Completing proofs**, to assure that the subtleties and details are captured;
- ▣ **Reinforcing notation and symbology that are particularly used in Mathematical Music Theory**, so that they become natural for those students interested in continuing with the area of study.

Chapter 4, in some way, tells a story

- ▣ The first section, **Musical Antecedents**, considers the relation between the study of pitch frequencies and the study of pitches from the perspective of structure.
- ▣ The notation used in **Mathematical Music Theory** is introduced;
- ▣ The notation is related to the basic concepts in Music for those that only have the mathematical knowledge, and the basic concepts of Group Theory for the musicians (**chromatic scale - \mathbb{Z}_{12} ; the formal definition of chord as a subset of the power set of \mathbb{Z}_{12} ,etc.)**)

In the second section:

- ▣ the “T” (transposition) and “I” (inversion) transformations are studied and defined in formal mathematical terms;
- ▣ Geometric representations are offered;
- ▣ The relations between T and I as mathematical objects are explored and propositions are proved, building up to the “climax”, where:
- ▣ It is formally proved that the “set” of these 24 transformations form a group of transformations under composition.

In the third section:

- ▣ The same thing is done in the third section with the “P” (parallel), “L” (leading tone) and “R” transformations.
- ▣ They are mathematically defined;
- ▣ Another type of geometric representation is offered (the *Tonnetz*, a particular representation in the Neo-Riemannian Theory);
- ▣ The relations between the P, L and R transformations are analyzed and propositions are proved until arriving at the theorem where it is also shown that, seen as objects, they form a group under composition.

- ▣ In section 4.4 an isomorphism between the TI and PLR groups is constructed;
- ▣ In section 4.5, it is seen that these two groups are dual, that is, each one is the centralizer of the other as subgroups of the symmetry group (permutations, that are also transformations) of the set of the 24 major and minor chords.
- ▣ Musical examples where the duality can be checked are also given here.

- ▣ This chapter is based on the article by Crans, Fiore and Santyedra and on Janine du Plessis's Masters thesis;
- ▣ An important contribution of Ms. du Plessis was the **extremely detailed formalization of the aspects that have been mentioned.**
- ▣ **This formalization is transferred to the nature of the exercises.** At the same time, the exercises are reasonable and formulated in pedagogical terms.