

# Proposal for 4030/6030 course 'Mathematical Music Theory'

**Statement.** Questions about variation, similarity, enumeration, and classification of musical structures have long intrigued both musicians and mathematicians. Mathematical models can be found for almost all levels of musical activities from composition to sound production by traditional instruments or by digital means. Modern music theory has been incorporating more and more mathematical content during the last decades. One example is the application of methods from algebraic combinatorics to the classification of different objects in music theory. Some of these objects are intervals, chords, tone-rows, motives, etc. The areas of mathematics that are currently active in music theory and will be included in the course are: Group Theory and other notions from modern algebra; combinatorics and enumeration; similarity and distance measures; theory of algebraic combinatorics on words; concepts from algebraic topology, and category and topos theory, which have served as the basis for the development of the software Rubato Composer and, presently, as the basis for a theory of creativity for music cognition.

During the past approximately 40 years, mathematical music theory has arisen in music departments, mathematics departments and computer science departments in the United States, Europe and Latin America. In music departments, the need to analyze the patterns of modern composition that could not be described by the tools of traditional music theory, gave way to musical set theory, where the use of group theory and other modern mathematical structures is as common as the application of mathematics in engineering or chemistry departments. On the other hand, mathematicians have found stimulating relations within applications of mathematics to established musical relations, which have led to interesting open problems in mathematics. Structures such as maximally even sets, developed in the context of mathematical music theory, have found applications in the Ising model in physics, for example. In this course, students will have the opportunity to see how their accumulated knowledge of abstract ideas can be applied to an important human activity, as well as reinforcing their dexterity in mathematics itself, while being exposed to modern theories in mathematics, music, and computer science.

## **Goals and Objectives.**

- (1) Critical understanding of the use of specific aspects of group theory, combinatorics, algebraic combinatorics on words, similarity measures, topology and geometry, and category theory in the analysis of general objects of music (scales, chords, rhythmic patterns) as well as specific applications

(development of software, music cognition, analysis of pieces from different time periods and genres).

- (2) Enrich students' repertoire of mathematical tools and techniques that are not usually covered in the core courses of the major. For example, in group theory students might need to study the semi-direct product, in word theory they will be exposed to Christoffel and Sturmian words and morphisms, and they will learn some of the basic constructions of category and topos theory, such as limits, colimits, subobject classifier, functors, natural transformations and Yoneda's lemma. All of this in the context of applications to musical objects and of some algorithmic procedures.
- (3) Provide a course environment that combines a deepening of knowledge from previously studied mathematical areas, with a novel experience in an interdisciplinary application of the abstract area.

### Learning Outcomes.

- Students will learn how to approach interdisciplinary applications of the abstract knowledge they have acquired in their study of mathematics;
- Students will understand how group theory notions are used in mathematical music theory, and will apply this understanding when carrying out analysis of musical objects;
- Students will study group theory constructions such as semi-direct product;
- Students will study concrete examples of how these constructions are applied to mathematical music theory. For example, the 288-element group of *uniform triadic transformations*, which is a recent development in transformational theory, and is isomorphic to  $\mathbb{Z}_{12}^2 \rtimes \mathbb{Z}_2$ , or *signature transformations* which are isomorphic to the cyclic group of 84 elements.
- Students will study basic notions of algebraic combinatorics on words;
- Students will study Christoffel and Sturmian words and morphisms, and will see how they are applied to the analysis of musical objects;
- Students will learn concepts from algebraic topology, such as simplices and nerve, and how they are related to topological spaces such as the Möbius strip. This can be illustrated in the context of

- applications of symmetries to a generalization of musical key modulation;
- Students will learn some of the basic constructions of category and topos theory, such as limits, colimits, subobject classifier, functors, natural transformations and Yoneda's lemma;
  - Students will see how the functorial approach is used in the software Rubato Composer;
  - Students will learn how they can develop "rubettes" in the context of Rubato Composer
  - Students will study distance and similarity measures in music and how they are used in music cognition.

### **Assessment of Learning Outcomes.**

Problems based on the learning objectives will be assigned on a regular basis and may appear in a variety of contexts:

- Classroom discussion provides an indication of the students' understanding of newly presented topics, of old material, and of their ability to relate new topics to old ones. Class meetings may involve a combination of lecture, questions and discussion, and small group work.
- Homework exercises related to the material that is being taught will be assigned throughout the course.
- In-class quizzes are essential to encourage students to work harder and to have a better understanding of the material. Quizzes assess students' ability to solve relatively simple problems.
- In-class exams give students the opportunity to demonstrate their ability to work on more complicated problems and to demonstrate their understanding of fundamental concepts.
- Take-home assignments for using Rubato Composer are designed to assess the students' ability to use this specialized computational tool.

### **Possible Textbooks.**

1. Mazzola, G. *The Topos of Music: Geometric Logic of Concepts, Theory and Performance*, Birkhäuser Basel, 2003, ISBN 9783764357313 (I am a contributor to the book, M.M.)

Lluis-Puebla, E., Mazzola, G. & Noll, T. *Perspectives in Mathematical and Computational Music Theory*, EpOs Osnabrück, 2004, ISBN 9783923486571. (I have a chapter in the book, M.M.)

### **Staffing.**

M. Montiel

### **Offering Schedule.**

The course will be taught once every two years.

**PROPOSED SYLLABUS**  
**Mathematical Music Theory**  
**4030/6030**

**Content:** You will study specific aspects of group theory, combinatorics, algebraic combinatorics on words, similarity measures, topology and geometry, and topos theory that are used in the analysis of general objects of music (scales, chords, rhythmic patterns) as well as in specific applications (development of software, music cognition, analysis of pieces from different time periods and genres). You will acquire a repertoire of mathematical tools and techniques that are not always covered in the core courses of the major. For example, in group theory you will need to study the semi-direct product, in word theory you will be exposed to Christoffel and Sturmian words and morphisms, and you will learn some of the basic constructions of category and topos theory, such as limits, colimits, subobject classifier, functors, natural transformations and Yoneda's lemma. All of this in the context of applications to musical analysis and of some algorithmic procedures in the software Rubato Composer.

**Textbook:** G. Mazzola, *The Topos of Music: Geometric Logic of Concepts, Theory and Performance*, Birkhäuser Basel, 2003, ISBN 9783764357313 (I am a contributor to the book, M.M.)

**Related Readings:**  
**Reading Material:**

Agustín, O., du Plessis, J., Lluís-Puebla, E. & Montiel, M. (2009). *An Introduction to Group Theory with Applications to Mathematical Music Theory*. Bookboon Ventus publishing Aps. ISBN: 978-87-403-0324-7. Online at: <http://bookboon.com/en/textbooks/mathematics/an-introduction-to-group-theory>

Johnson, Timothy, *Foundations of Diatonic Theory: A mathematical approach to Music Fundamentals*, The Scarecrow Press, 2008.

Keith, Michael, *From Polychords to Polya: Adventures in Musical Combinatorics*, Vinculum Press, Princeton, N.J, 2001.

Milmeister, Gérard, *The Rubato Composer Music Software: Component-Based Implementation of a Functorial Concept Architecture*, Springer-Verlag, 2009, ISBN: 978-3-642-00148

Assorted articles that have been key in the development of the field of Mathematical Music Theory.

**Prerequisite:** Grade of C or higher in MATH 3000.

**Corequisite:** Math4441/6441

**Administrative Drop Policy:** Students who did not take the required prerequisite or do not attend the class regularly during the first two weeks will be administratively dropped.

Grading will be based on:

- 1) 2 in class exams and the programmed two and a half hour final exam;
- 2) 2 quizzes
- 3) Take home assignments that will include exercises and Rubato Composer computer tasks;
- 4) A Final Project, which will be based on one of the areas you find of particular interest, and that will take the form of a mathematical paper and, if time permits, a 10 minute presentation.

Graduate students will be expected to answer some extra problems on the exams.

The exams will be 40% of the final grade, the final exam will be 20% of the final grade, the quizzes will be 10% of the final grade, the assignments will be 10% of the final grade, and the project will be 20% of the final grade.

97 - 100 = A+

93 - 97 = A

90 - 92 = A-

87 - 90 = B+

83 - 87 = B

80 - 82 = B-

77 - 79 = C+

70 - 76 = C

60 - 69 = D

0- 59 = F

### Tentative Calendar

Date	Topic	Complementary Sources	Evaluations
<b>Week 1</b>	<i>Spatial Relations and Musical Structures; Interval Patterns and Musical Structures</i>	Johnson, Timothy, <i>Foundations of Diatonic Theory: A mathematical approach to Music Fundamentals</i> , The Scarecrow Press, 2008	
<b>Week 2</b>	<i>Continuation</i>		
<b>Week 3</b>	<i>Applications of Group Theory: Transformational Theory</i> T/I, PLR groups; Contextual groups; Signature Transformations; Uniform Triadic Transformations	<ul style="list-style-type: none"><li>• Agustín, O., du Plessis, J., Lluís-Puebla, E. &amp; Montiel, M. <i>An Introduction to Group Theory with Applications to Mathematical Music Theory</i>, <b>Chapter 4.</b></li><li>• Fiore- <i>Generalized Contextual Groups</i></li><li>• Hook, J. <i>Signature Transformations; Uniform Triadic Transformations</i></li></ul>	
<b>Week 4</b>	<i>Applications of Group Theory Cont'd; Duality of Contextual Groups.</i>	Same as above.	Quiz 1
<b>Week 5</b>	<b>Symmetry and Transformations in Music</b>	<i>Assorted materials on finite isometry groups, frieze and wallpaper groups;</i>  Hart, Vi, <i>Symmetry and Transformations in the Musical Plane, Proceedings of Bridges 2009: Mathematics, Music, Art, Architecture, Culture (2009) 169–176</i>	

<b>Week 6</b>	<i>Maximal Evenness in Rhythm; Chains, Necklaces and Partitions; Mathematical Features for Recognizing Preference in Sub-Saharan Rhythms.</i>	Keith, Michael, <i>From Polychords to Polya: Adventures in Musical Combinatorics</i> , <b>Chapter 3</b> . Vinculum Press, Princeton, N.J, 2001.  <i>Assorted Articles.</i>	
<b>Week 7</b>	<i>Similarity Measures; Music Cognition</i>	<ul style="list-style-type: none"> <li>• J. Miguel Díaz-Báñez Giovanna Fariguy Francisco Gómez, David Rappaport, Godfried T. Toussaint - <i>El Compas Flamenco: A Phylogenetic Analysis</i>;</li> <li>• Schum, <i>Testing Models of Musical Contour Similarity</i>.</li> <li>• Sethares, W.- <i>Distance Topologies</i></li> <li>• Buteau, C., Mazzola, G. <i>From Coutour Similarity to Motivic Topologies</i>.</li> <li>• Norgaard, M., Spencer, J. Montiel, M. <i>Testing Cognitive Theories: a Probabilistic Based Algorithm for Melody and Rhythm in Jazz Improvisation</i>.</li> </ul>	
<b>Week 8</b>	<i>Continuation</i>		<b>Exam 1</b>
<b>Week 9</b>	<i>Rubato Composer; Forms and Denotators</i>	Milmeister, Gérard, <i>The Rubato Composer Music Software: Component-Based Implementation of a Functorial Concept Architecture</i> , Springer-Verlag, 2009, ISBN: 978-3-642-00148 <b>Chapters 1-5, 15,16.</b>  <b>Download Software</b>	
<b>Week 9</b>	<i>Continuation: Rubato Composer</i>		<b>Homework Assignment</b>
<b>Week 10</b>	<i>Algebraic Combinatorics on Words</i>	<ul style="list-style-type: none"> <li>• Noll, T. &amp; Clampitt, D. <i>Modes, The Height-Width Duality, and Handsin's Tone Character</i></li> <li>• Berstel, Lauve, Reutenauer, Saliola-<i>Combinatorics on Words, Christoffel Words and Repetitions in Words</i> (Book)</li> <li>• Berthé- <i>Sturmian Sturmiian Involution</i></li> </ul>	

		<ul style="list-style-type: none"> <li>Noll, T., Montiel, M. <i>Glarean Revisited</i></li> </ul>	
<b>Week 11</b>	<i>Continuation Algebraic Combinatorics on Words</i>		
<b>Week 12</b>	<i>Continuation</i>		Exam 2
<b>Week 13</b>	<i>Geometry and Topology in Music Theory</i>	<ul style="list-style-type: none"> <li>Muzzolini D., <i>Musical Modulation by Symmetries.</i></li> <li>Tymocko, D., <i>A Geometry of Music</i></li> </ul>	
<b>Week 14</b>	<i>Continuation</i>		Quiz 2
<b>Week 15</b>	<i>15 minute presentations</i>		
<b>Final Exam</b>			

***Missed exam policy:** There will be **no makeup exams except in extreme cases that MUST be justified.** Missed exams will receive a grade of zero. Any conflicts must be worked out with me **ahead of time.***

***Attendance policy:** Come to class on time and do not leave early. Anything else is rude and disruptive. **If for any reason the University is closed on an exam or quiz day, the exam or quiz will be given at the next regularly scheduled class meeting.** A student is responsible for all material covered in class, whether he or she attended the class. The instructor may drop a student from the roll for exceeding three class absences. **Any student who does not complete the course for any reason must officially withdraw from the course or will be awarded a grade of F.***

***Classroom Policy:** Please refrain from using cell phones and laptops in the classroom.*

The following policy applies to all courses in the Department of Mathematics and Statistics:

*If you do not attend class during the first week you will be administratively dropped.*

*Administrative withdrawal--Students with excessive absences or students without the necessary prerequisites will be administratively withdrawn.*

*Copied exam papers will not be accepted and will receive a grade of zero.*

*See the University's policy on Academic Honesty:*

[www.gsu.edu/~wwwdos/codeofconduct\\_conpol.html](http://www.gsu.edu/~wwwdos/codeofconduct_conpol.html)

*Disruptive behavior: See the University's policy on disruptive behavior:*

[www.gsu.edu/~wwwsen/minutes/2002-2003/disrupt.html](http://www.gsu.edu/~wwwsen/minutes/2002-2003/disrupt.html)